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A Parametric Study on A Curvature Approximation Based on Interaction Diagrams of Concrete Columns Exposed to An ISO 834 Standard Fire

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Abstract

Fire has a significant influence on concrete structures and members. A concrete column, compared to other structural members, has most often to cope with vertical forces and bending moments from slabs and beams. Therefore, it is important to investigate the fire resistance of concrete columns. In fact, simplified methods are often adopted to evaluate the capacity of concrete columns. In the *fib* Model Code 2010, a curvature approximation is introduced. However, this method has not been validated to be used in case of fire. Hence, a parametric study is performed in this paper to investigate the application of this method.

As a first step of this paper, a curvature approximation formula used at ambient temperature is introduced. As this formula is based on the curvature and in order to verify the application, a numerical tool that can obtain the bending moment and curvature relationships is presented and validated. Further, an ISO 834 standard fire is adopted. Finally, parameters like dimensions, the reinforcement ratio and the slenderness ratio are investigated. Comparing the effect of dimensions, the reinforcement ratio as well as the slenderness ratio, it is concluded that only the slenderness ratio has a significant influence on the column capacity with the proposed formula. The difference between the deflections obtained with the simplified method and the numerical values increase in function of the slenderness ratio (in case of the same axial load). However, this method is proven to be easy-to-use and safe for the prediction of the fire resistance of concrete columns.

Keywords: concrete columns; standard fire; curvature approximation; fire resistance

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1. Introduction

In order to investigate the mechanical behaviour of reinforced concrete columns, simplified methods are often adopted. The curvature method, based on the effective length l_0 and an estimated maximum curvature, is primarily suitable for isolated members with constant normal force and a defined effective length [1]. Robinson et al. [2] provided a more fundamental basis than the curvature method, i.e. the so-called curvature-based approach in line with the major steps of the "sinusoidal total eccentricity method". This method is also described by Marí and Hellesland [3] with some explanations and application conditions. Further, in the *fib* Model Code 2010 [4], an approximation of the maximum design curvature is presented. However, the results obtained with the curvature method are over-conservative comparing with experimental results [5]. Furthermore, the curvature approximation has not validated to be used in case of fire. Therefore, it is important to investigate the application of the curvature approximation method in case of fire.

This paper proposes a numerical calculation tool in order to verify the application of the curvature approximation in case of fire. Parameters like dimensions, the reinforcement ratio and the slenderness ratio are investigated in case of an ISO 834 standard fire.

2. Curvature approximation method

In the *fib* Model Code 2010 [4], an approximation of the maximum design curvature is presented. The design value of the bending moment is:

$$M_{Ed} = -N_{Ed} e_d \tag{1}$$

where N_d is the axial load

e_d is the maximum eccentricity, being the maximum distance between the compression resultant and the deformed axis of the compression member;

$$e_d = e_i + e_{1d} + e_{2d} \tag{2}$$

where e_i is the eccentricity due to imperfections

e_{1d} is the first-order eccentricity, $e_{1d} = -\frac{M_{1d}}{N_d}$; M_{1d} is the first-order moment

e_{2d} is the eccentricity due to the deformation of the compression member, $e_{2d} = \frac{\kappa_d l_0^2}{c_0}$; c_0 is the integration factor accounting for the curvature distribution along the member, κ_d is the maximum design curvature

$$\kappa_d = \frac{\varepsilon_{sd} - \varepsilon'_{sd}}{h-2c} \tag{3}$$

where ε_{sd}, ε'_{sd} are the strains of the reinforcing bars at the top and the bottom layers

h is the height of the cross-section

c is the cover thickness

The maximum design curvature may be obtained with $\varepsilon_{sd} = f_{yd}/E_s$ and $\varepsilon'_{sd} = -f_{yd}/E_s$.

Based on Eq. (3) for the maximum design curvature of point B shown in Fig. 1, the second method provides an equation to obtain a more accurate value of the maximum design curvature for any point between A and B:

$$\kappa_d = K_r \cdot \frac{\varepsilon_{sd} - \varepsilon'_{sd}}{h-2c} = \left(\frac{n_u - n_d}{n_u - n_{bal}}\right) \cdot \frac{\varepsilon_{sd} - \varepsilon'_{sd}}{h-2c} \tag{4}$$

where $K_r = \frac{n_u - n_d}{n_u - n_{bal}}$ is a correction factor depending on the axial load

$n_u = 1 + \omega$

$\omega = \frac{A_s f_{yd}}{A_c f_{cd}}$

$n_d = \frac{N_{Ed}}{A_c f_{cd}}$

n_{bal} is the value of n at maximum moment resistance; $n_{bal} \approx 0.4$ (point B is shown in Fig. 1)

Assuming the distance between the reinforcing bars as 0.9h and the reinforcement yields at both sides, Eq. (4) can be simplified as:

$$\kappa_d = \left(\frac{n_u - n_d}{n_u - n_{bal}}\right) \cdot \frac{\varepsilon_{yd}}{0.45h} \tag{5}$$

Eq. (5) is based on interpolation adopting a linearized interaction diagram shown in Fig. 1. It is worth pointing out that at point B, the reinforcement yields at both sides of the column at ambient temperature, so that the curvature is $\kappa = \frac{\varepsilon_{yd}}{0.45h}$. Considering the curvature $\kappa = 0$ at point A, the curvature in point C can be obtained by a linear interpolation from Eq. (4). It is worth pointing out that at point B, the reinforcement yields at both sides of the column at ambient temperature, so that the curvature is $\kappa = \frac{\varepsilon_{yd}}{0.45h}$. Considering the curvature $\kappa = 0$ at point A, the curvature in point C can be obtained by a linear interpolation from Eq. (5).

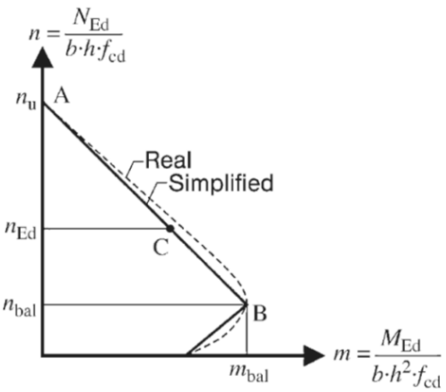

Fig. 1. Simplified representation of interaction curves

3. Calculation tool

A cross-sectional numerical calculation tool is proposed to calculate the combined effect of an axial force (N) and bending moment (M) on columns, taking into account material strength reduction and thermal strains in case of fire. This calculation model takes the material model (siliceous aggregate concrete and steel) of EN 1992-1-2 [6] as a basis for both the thermal analysis and structural analysis. Assumptions are made for a simplified calculation: 1) concrete has no tensile strength; 2) plane sections remain plane 3) the temperature of the steel is assumed to be uniform over the steel cross-section. This numerical method is based on a cross-sectional calculation for both the thermal analysis and the structural analysis and the calculation tool has been introduced and validated in Wang [5].

4. Examples

A column with a cross-section 300 mm × 300 mm and a cover thickness 25 mm is analysed for different reinforcement ratios of 0.1, 0.5 and 1.0 in case of an ISO 834 fire with a duration of 0 min (ambient temperature), 30 min, 60 min and 90 min. Interaction curves of columns at ambient temperature are obtained in Fig. 2, where $n' = \frac{N_{Ed}}{bhf_c}$, $m_x = \frac{M_{Ed}}{bh^2 f_c}$, b is the width of the cross-section, h is the height of the cross-section.

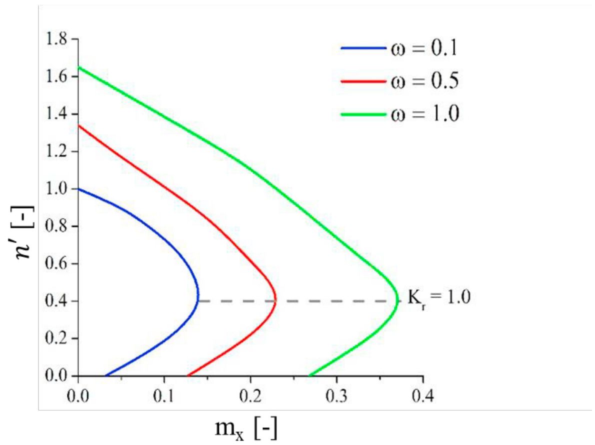

Fig. 2. Interaction curves of columns at ambient temperature for reinforcement ratios 0.1, 0.5 and 1.0

On the basis of Fig. 2, it is seen that n_{bal} can be considered as 0.4 also for columns with different reinforcement ratios. Further, interaction curves are obtained for the same columns, considering $n' = \frac{N_{Ed}}{bhf_c}$ and $m = \frac{M_c+M_s}{0.7(A_c f_{cd} + A_s f_{yd})h}$ as explained in Eurocode 2 (2004) that 0.7 as a reduction factor for the design load in case of fire. It is concluded that the value of n_{bal} as well as n' (overall) decreases with fire duration. As a simplification, it is observed that the interaction curves for values of n larger than n_{bal} can still be approximated by a linear relationship.

In order to check the accuracy of the simplified method, the maximum curvature is investigated using the calculation tool. First, a column with a cross-section of 300 mm × 300 mm, a Ø32 bar in each corner and a cover thickness of 25 mm is chosen. The material properties are: 20°C concrete compressive strength f_{ck} = 55 MPa, reinforcement yield strength f_y = 500 MPa and Young's modulus of steel $E_s = 2\times10^5$ N/mm². The moment-curvature diagrams based on the tool at ambient temperature in case of different axial loads are illustrated in Fig. 3, where $n' = \frac{N_{Ed}}{bhf_c}$, M is the bending moment capacity, χ is the corresponding curvature, point B is the peak point which represents the maximum design curvature and $(\varepsilon_1, \varepsilon_2)$ are strains of the reinforcement at point B.

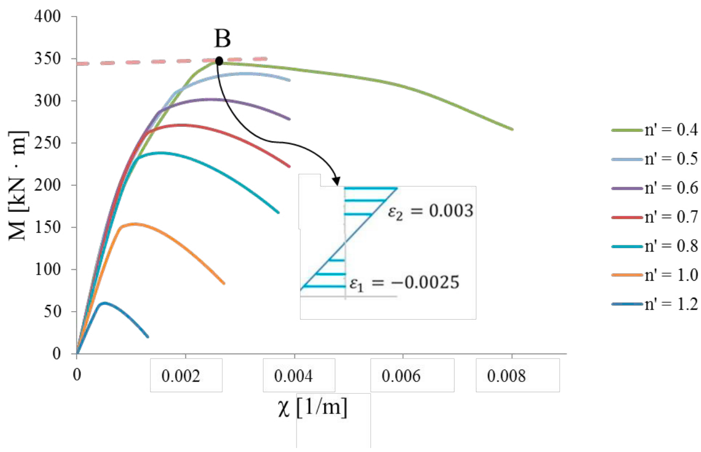

Fig. 3. The bending moment-curvature diagrams of the column at ambient temperature and the strain distribution at point B at ambient temperature

Fig. 3 shows the relationship between the bending moment and the curvature in case of different axial loads. It is worth mentioning that at the peak of the curve in case of n' = 0.4, the strain of the reinforcing bars in the tensile zone does not equal the limit strain because at both sides the reinforcing bars not exactly yield in case of n' = 0.4. However, as a simplification, $n_{bal} = 0.4$ may be used.

Since our numerical calculation method is based on the strain distribution of the cross-section which has the same basis as the method provided in *fib* model code 2010 [4] using Eq. (4) or a more simplified formula Eq. (5), the comparison is made in order to develop this method to the application of fire (Table 1).

Table 1. Comparisons of the maximum design curvature obtained with the simplified formula, interaction diagrams based calculation and the numerical method

n' \ Curvature [1/m]	Simplified formula (Eq.(5)) (1)	Simplified formula (Eq.(4)) (2)	Numerical values (3)	$\frac{(3)-(1)}{(3)}$ (%)	$\frac{(3)-(2)}{(3)}$ (%)
0.4	0.019	0.025	0.025	24	0
0.5	0.017	0.023	0.031	45	26
0.6	0.015	0.02	0.025	40	20
0.7	0.013	0.017	0.019	32	11
0.8	0.011	0.014	0.015	27	7
1	0.007	0.009	0.011	36	18
1.2	0.003	0.004	0.005	40	20
1.34	0	0	0	—	—

It is observed that Eq. (5) gives very conservative results for the maximum design values. The difference at ambient temperature could reach 45% in this case. Comparing with the simplified formula Eq. (5), the prediction based on Eq. (4) is closer to the numerical values. However, the prediction is still conservative. Therefore, an improved formula is derived for the calculation of the maximum design curvatures (which will be furthermore extended to the case of fire).

In order to find a simplified way to determine the maximum design curvature at ambient temperature as well as in case of fire, the strain distribution obtained with the numerical tool is investigated. The aforementioned column which has been investigated at ambient temperature is further adopted in case of an ISO 834 standard fire. Considering the effect of a fire duration of 30 minutes, strain distributions along the central axis in case of n = 0.3, 0.5, 0.7 and 0.9 are illustrated in Fig. 4 in order to figure out the applicability of Eq. (3).

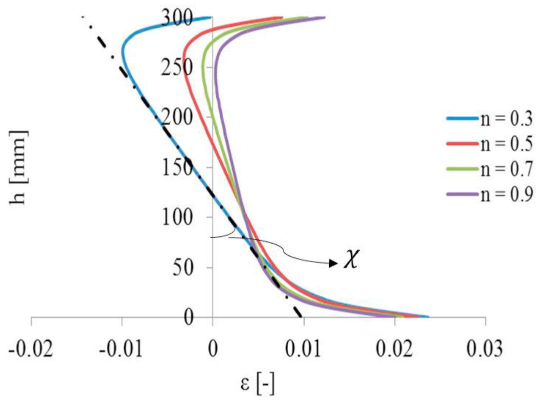

Fig. 4. The strain distribution of the maximum design curvature of columns exposed to an ISO 834 fire of a duration of 30 minutes in case of the axial load n = 0.3, 0.4, 0.5, 0.7 and 0.9

Fig. 4 shows the strain distribution of the maximum design curvature of columns in case of a fire duration of 30 minutes. It is seen that the strain distribution is nonlinear due to the effect of thermal strain. However, the slope of the strain diagram in the central part of the cross-section tends to be a constant in case of a 30-minutes fire. That means that with respect to the maximum design curvature in case of fire, an assumption of a linear strain distribution for the core of the cross-section is feasible at least for a small fire duration. As a basic calculation principle, the maximum design curvature in case $n \approx 0.4$ where the strain at the centroid of the cross-section equals 0 is used at ambient temperature. Hence, the same calculation condition which is required for n_{bal} is proposed in case of fire. As such, an improved formula, developed from Eq.(5), is proposed:

$$\kappa_d = \left(\frac{n_u - n_d}{n_u - n_{bal}}\right)\chi_{bal} \tag{6}$$

Next, the same column is investigated in case of a fire duration of 60 minutes, 90 minutes and 120 minutes. Based on the calculation with the numerical tool, $\chi_{bal} = 0.080$ 1/m and $n_{bal} = 0.4$ are obtained in case of the fire duration 60 minutes while $\chi_{bal} = 0.096$ 1/m and $n_{bal} = 0.3$ hold in case of the fire duration 90 minutes and $\chi_{bal} = 0.094$ 1/m and $n_{bal} = 0.3$ in case of the fire duration 120 minutes. The maximum design curvature obtained with Eq. (6) and the numerical values are shown in Table 2.

Table 2. Comparisons of the maximum design curvature obtained with Eq.(6) and the numerical method in case of fire

Curvature [1/m] / n'	Simplified calculation based on (Eq.(6)) (1)				Numerical values (2)				$\frac{(2)-(1)}{(2)}$(%)			
	Fire duration [min]				Fire duration [min]				Fire duration [min]			
	30	60	90	120	30	60	90	120	30	60	90	120
0.3	—	—	0.096	0.094	0.084	0.1	0.096	0.094	—	—	0	0
0.4	0.06	0.078	0.077	0.062	0.06	0.078	0.08	0.066	0	0	4	6
0.5	0.053	0.065	0.058	0.031	0.054	0.07	0.06	0.042	2	7	3	26
0.6	0.045	0.052	0.038	—	0.046	0.06	0.042	—	2	13	10	—
0.7	0.038	0.039	0.019	—	0.038	0.044	0.02	—	0	11	5	—
0.8	0.03	0.026	—	—	0.032	0,032	—	—	6	19	—	—
0.9	0.023	0.013	—	—	0.026	0.018	—	—	12	28	—	—
1	0.015	—	—	—	0.018	—	—	—	17	—	—	—
1.1	0.008	—	—	—	0.008	—	—	—	0	—	—	—

Table 2 illustrates the maximum design curvature to be used in Eq. (6). The results calculated with Eq. (6) are more conservative than the ones from the numerical method. However, the difference is quite reasonable and safe enough to predict the deflection due to the second order effects as well as the bending moment capacity.

5. A parametric study

In order to verify the applicability of Eq. (6), parameters like dimensions, the reinforcement ratio as well as the slenderness ratio are investigated. Three groups of parametric variations are listed in Table 3. In the comparison of the three groups, the maximum design curvatures in case of an ISO 834 fire of a fire duration of 60 minutes are used.

Table 3. Parametric study for assessing the validity of Eq. (6)

Comparison Group No.	Dimensions [mm×mm]	Reinforcement ratio	Slenderness ratio
1	150 mm×150 mm 300 mm×300 mm 500 mm×500 mm	0.5	0
2	300 mm×300 mm	0.1,0.5,1.0	0
3	300 mm×300 mm	0.5	30,40,50

5.1. Effect of dimensions

In group 1, three columns of different cross-sections as listed in Table 3 are analysed. The maximum design curvatures obtained with Eq. (6) (designated 1) and the numerical tool (designated 2) in case of a fire duration of 60 minutes are shown in Table 4.

Table 4. Comparisons of the maximum design curvature obtained with Eq. (6) (1) and the numerical method (2) in case of a fire duration of 60 minutes considering the effect of different cross-sectional dimensions

Curvature [1/m]	Dimensions [mm×mm]								
	150 mm×150 mm			300 mm×300 mm			500 mm×500 mm		
n'	(1)	(2)	$\frac{(2)-(1)}{(2)}$(%	(1)	(2)	$\frac{(2)-(1)}{(2)}$(%	(1)	(2)	$\frac{(2)-(1)}{(2)}$(%
0.3	0.22	0.22	0	—	0.1	—	—	0.06	—
0.4	0.165	0.18	8	0.078	0.078	0	0.039	0.039	0
0.5	0.11	0.14	21	0.065	0.07	7	0.033	0.035	6
0.6	0.055	0.06	8	0.052	0.06	13	0.028	0.03	7
0.7	—	—	—	0.039	0.044	11	0.022	0.025	12
0.8	—	—	—	0.026	0.032	19	0.017	0.018	6
1	—	—	—	—	—	—	0.006	0.009	33

Table 4 indicates that the results obtained with Eq. (6) for all the three columns are close to the numerical values and all are on the safe side. Hence, it follows that the prediction is insignificantly influenced by the dimensions.

5.2. Effect of reinforcement ratio

In group 2, columns of different reinforcement ratios are calculated for the maximum design curvatures obtained with Eq. (6) (1) and the numerical tool (2) in case of a fire duration of 60 minutes and compared in Table 5.

Table 5. Comparison of the maximum design curvature obtained with Eq. (6) (1) and the numerical method (2) in case of a fire duration of 60 minutes considering the effect of different reinforcement ratios

Curvature [1/m]	Reinforcement ratio [-]								
	0.1 [-]			0.5 [-]			1 [-]		
n'	(1)	(2)	$\frac{(2)-(1)}{(2)}$(%	(1)	(2)	$\frac{(2)-(1)}{(2)}$(%	(1)	(2)	$\frac{(2)-(1)}{(2)}$(%
0.3	—	0.1	—	—	0.1	—	—	0.1	—
0.4	0.06	0.06	0	0.078	0.078	0	0.092	0.092	0
0.5	0.042	0.042	0	0.065	0.07	7	0.081	0.081	0
0.6	0.025	0.03	17	0.052	0.06	13	0.07	0.076	8
0.7	0.007	0.014	50	0.039	0.044	11	0.059	0.07	16
0.8	—	—	—	0.026	0.032	19	0.048	0.066	27
1	—	—	—	—	—	—	0.037	0.046	20

From Table 5, it is seen that only the prediction in case of the reinforcement ratio 0.1 and the axial load 0.7 is too conservative. For other cases, this simplified calculation is in good agreement with the numerical results.

5.3. Effect of slenderness ratio

Group 3 presents columns for different slenderness ratios. The maximum design curvatures of the cross-sectional calculation shown in Table 2 are adopted to calculate the second order eccentricity for the slenderness ratios 30, 40 and 50. As a result, the second order deflections calculated with Eq. (6) (1) are compared with results from the numerical tool (2) (Table 6).

Table 6: Comparisons of design deflections (m) obtained with Eq. (6) (1) and the numerical method (2) in case of a fire duration of 60 minutes considering the effect of slenderness ratios

Curvature [1/m] \ n'	Slenderness ratio [-]								
	30 [-]			40 [-]			50 [-]		
	(1)	(2)	$\frac{(1)-(2)}{(1)}(\%)$	(1)	(2)	$\frac{(1)-(2)}{(1)}(\%)$	(1)	(2)	$\frac{(1)-(2)}{(1)}(\%)$
0.4	0.053	0.044	17	0.095	0.073	23	0.148	0.109	25
0.5	0.044	0.041	7	0.079	0.068	14	0.123	0.1	19
0.6	0.036	0.035	3	0.063	0.056	11	0.099	—	—
0.7	0.027	0.026	4	0.047	—	—	0.074	—	—

Table 6 indicates that the difference between the simplified equation (2) and the numerical method increases with the slenderness ratio in case of the same axial load. This difference reaches a maximum of 25% in case of these three columns. The prediction calculated with Eq. (6) is most often in good agreement with the numerical value and is always on the safe side. Hence, it can be concluded that this improved formula can be adequately used in case of fire design.

6. Conclusions

Comparing the effect of dimensions, the reinforcement ratio as well as the slenderness ratio, it is concluded that only the slenderness ratio has a significant influence on the prediction of the second order effects with the proposed simplified formula. The difference between the deflections obtained with the simplified method and the numerical values increase in function of the slenderness ratio (in case of the same axial load). However, the simplified formula is proven to be easy-to-use and safe for the prediction of second order effects in columns exposed to fire.

Acknowledgements

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